Traffic lights Vs Traffic circles

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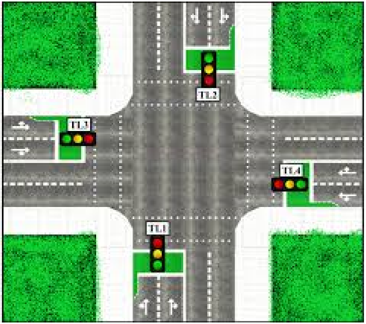
Introduction

(a) Traffic circle



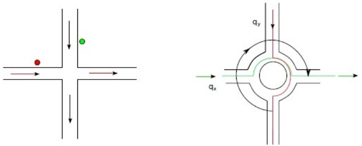
(b) Traffic lights

Objectives

- Primary aim: 'Facilitate' traffic flow through the intersection
  - reduce average time delays of individual drivers?
  - maximize flux through the intersection?
  - reduce driver stress?
- Safety (accidents)
- Costs

Observations

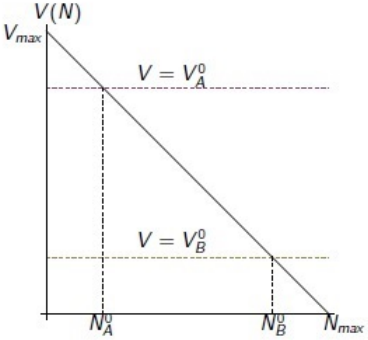
- Traffic circles: Traffic flow is continuous
- Traffic lights
  Traffic flow is less obstructed. Flow rate, when it occurs, is likely to be greater. Flow can be better regulated.
- Complicated problem: optimum solution
  Z(roundabout size, flux levels, number of streams, ...)
- Simpler problem



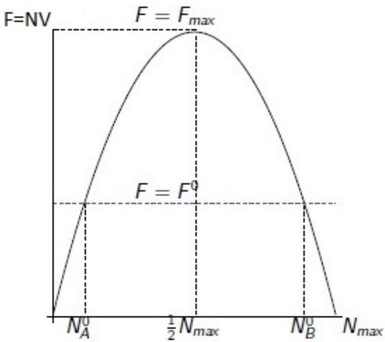
Traffic flow with two streams

Driver behaviour

N=traffic density (cars/km), V=traffic speed (km/hr), F=traffic flux (cars/hr)



$$V = V_{max}\left[1 - \frac{N}{N_{max}}\right].$$



$$F = V_{max}\left[1 - \frac{N}{N_{max}}\right]N.$$

Small density, high speed (supersonic) soln (N_A^0, V_A^0)
Large density, low speed (subsonic) soln (N_B^0, V_B^0)

Unsteady single lane traffic flow

• Car conservation

$$\frac{\partial N}{\partial t} + \frac{dF}{dN}\frac{\partial N}{\partial x} = 0 \quad (1)$$

with known initial and boundary conditions $N(0, x)$ and $N(t, 0)$.

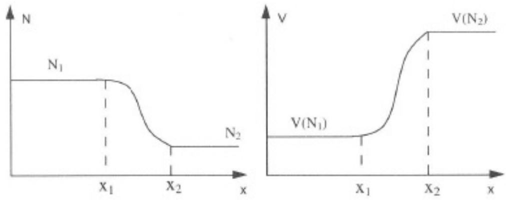
• When a general solution to (1) N remains constant along the path $x = X(t)$ with the total derivative

$$\frac{d}{dt}[N(X(t), t)] = 0,$$

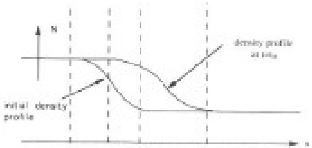
therefore

$$\frac{dX}{dt}(t) = \frac{dF}{dN}(N). \quad (2)$$

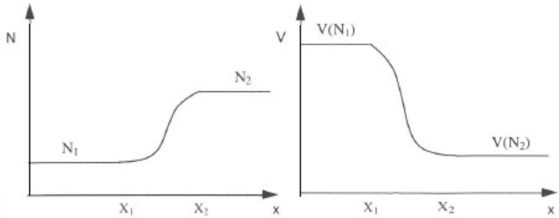
• Equation (2) represents signal speed (speed of information).



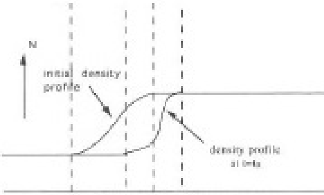
The initial density and velocity distributions.



Expansion



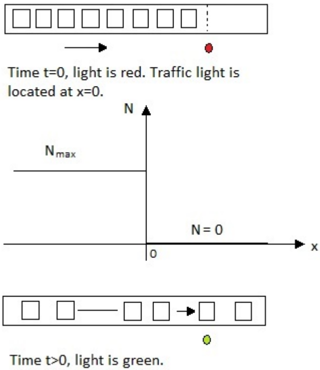
initial profiles.

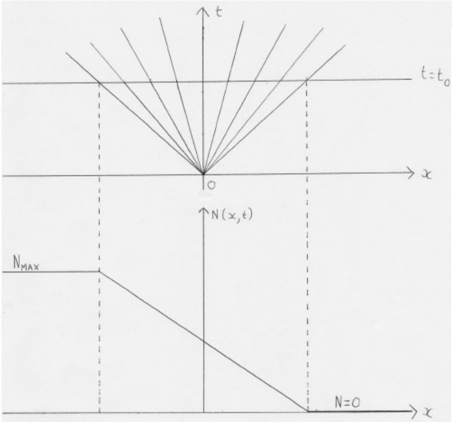


Shock

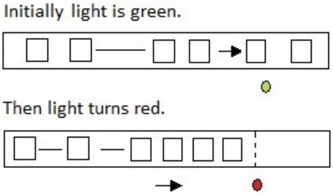
Traffic light turns green

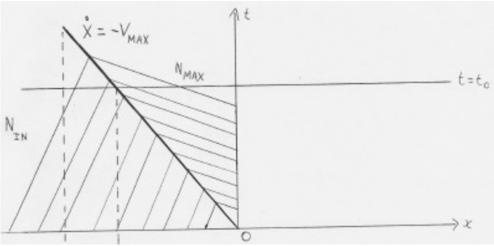
- Single lane, one-direction.

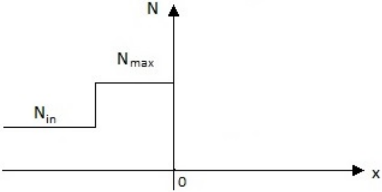




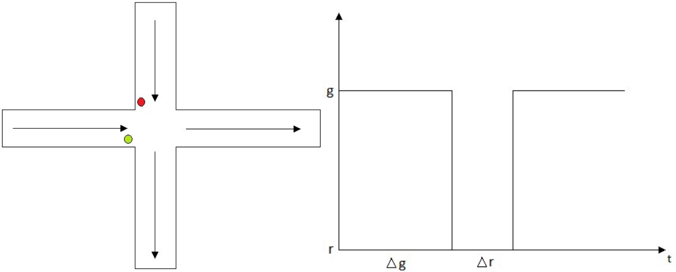
Traffic light turns red







Two single streams

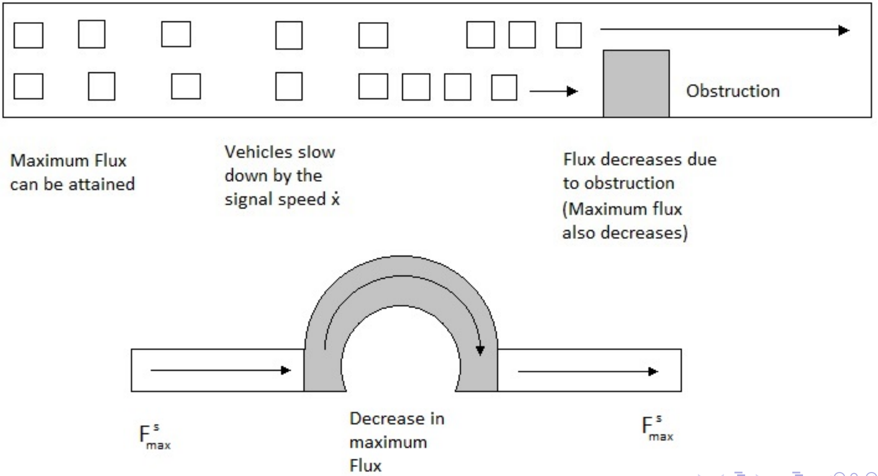


$$F_{tot} = F^x_{max}\Delta_x + F^y_{max}\Delta_y$$

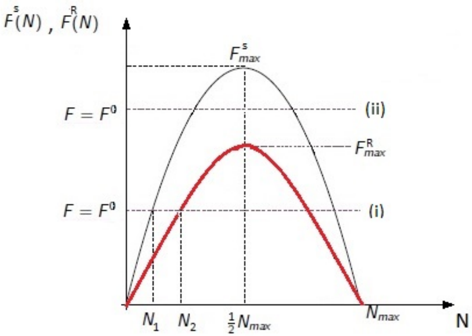
Average Flux

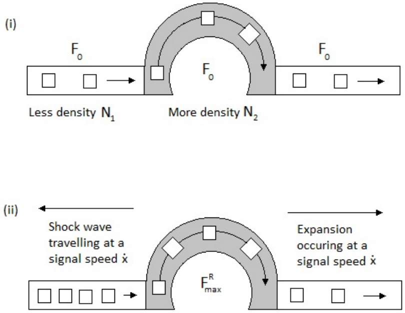
$$F_{Av} = \frac{\Delta_g F^x_{max} + \Delta_r F^y_{max}}{\Delta_g + \Delta_r}.$$

Simple obstruction model



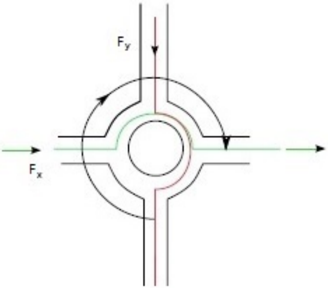
Analysis





- (i) Case 1: $F_0 < F^R_{max}$
- (ii)Case 2: $F_0 > F^R_{max}$

Two streams



$$F^{x}_{max} + F^{y}_{max} = F^{R}_{max}$$

Illustrative Example 1

$F^{R}_{max} = 60$ cars/minute

$F_x = 30$ cars/minute

$F_y = 20$ cars/minute

$\therefore$ Flow is continuous (No car build up)

$F_x + F_y < F^{R}_{max}$

Illustrative Example 2

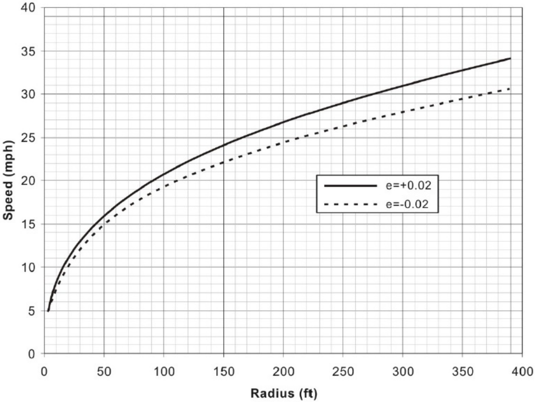
$F^{R}_{max} = 60$ cars/minute

$F_x = 40$ cars/minute

$F_y = 60$ cars/minute

$\therefore$ Shock waves result since $F_x + F_y > F^{R}_{max}$

Effect of the radius on flow



Velocity profiles plotted against the radius for the superelevation $e = -0.02$ and $e = 0.02$.

Maximum flux attainable : F^R_{max}, $\frac{\Delta_g F^x_{max} + \Delta_r F^y_{max}}{\Delta_g + \Delta_r}$.

- Traffic circle (roundabout 'R')
  - Flow is continuous
  - The radius is shown to have a big impact on the flow rate and therefore the maximum flux, however building a big roundabout as opposed to a small one is costly.
  - Safety issues: Entry depends a lot on driver behaviour.
- Traffic lights
  - Flow rate, when it occurs, is likely to be greater.
  - Drivers' stress: Different cases of different flux from two streams can be considered in determination and alleviation of stress.
  - Safety issues: flow can be better regulated.